RADIATION EFFECTS ON CONVECTION FLOW OF HEAT GENERATING FLUID IN A POROUS VERTICAL CHANNEL WITH VELOCITY SLIP AND TEMPERATURE JUMP

Supported by tetfund TERTIARY EDUCATION TRUST FUND

A.B. Disu[1], S.A. Amoo[2] and S.O. Ajibola[1]

[1]School of Science and Technology, National Open University of Nigeria, Victoria Island, Lagos

[2]Department of Mathematics & Statistics, Federal University Wukari, Nigeria

adisu@noun.edu.ng and drsikiruamoo@gmail.com;

Abstract: This study was conducted to investigate unsteady natural convection flow of heat generating fluid in a porous vertical channel with velocity slip and temperature jump in the presence of thermal radiation. The governing equations of fluid flow and heat transfer were separated into steady and periodic parts. Exact solutions were obtained for velocity, temperature, skin friction and rate of heat transfer. The effects of radiation and other fluid parameters involved in the fluid flow were highlighted and discussed.

Keywords: Micro flow devices, thermal radiation, porous vertical channel.

Introduction

The study of unsteady natural convective flow of viscous incompressible fluid in a channel has gained considerable attention of researchers in the last few decades. The interest of the study stems from the applications in designing cooling of electrical and electronic component, automatic control system. The theoretical and practical significance of designing of cooling system were discussed by Wang (1988). In view of this, Jha and Ajibade (2012) studied the effects of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions. The authors pointed out that positive value of viscous dissipation parameter suppressed the Strouhal number number. Also, Jha and Ajibade (2009) considered free convective flow of heat generating/absorbing fluid between vertical porous plates with periodic heat input. The authors reported that introduction of suction/injection suppressed influence of heat sink. All the investigations were conducted to improve the cooling of system designs.

In all the above mentioned studies, the effects of velocity slip and temperature jump were neglected. However, with influx of micro devices in the last few decades for household and industrial usages, exclusion of velocity slip and temperature jump in the studies may not be efficient and effective remove heat generated withinmicro-flow devices (Adesanya, 2015). The most efficient way of high heat flux removal from small area ismicro channels were investigated by Hung and Ru (2006) and Dharaiya and Kandlikar (2012). The micro channels are referred to as micro-flow devices such as turbines, pumps, heat pipes and mixers. The devices are very important in cooling of electrical andelectronic components design, chemical analysis, biotechnology, medical diagnosis and surgery. (Vafai and Zhu, 1999; Vafai and Khaled, 2005)

The flow regimes in micro-devices can be classified using Knudsen number.Knudsen number is a measure of the validity of the continuum model. The Navier-Stokes equations assumed that the continuum flow with the no-slip conditions at $Kn < 0.001$. The continuum valid for the slip boundary conditions at $0.01 < Kn < 0.1$. The regimes of flow within $0.01 < Kn < 0.1$ is called slip flow regime. The higher values of Knudsen number, the Navier-Stokes assumption is not valid (Dehghan and Basirat, 2014). The study of fluid flow in micro-devices is regarded as a continuum flow in a channel. Modeling of flow and heat transfer in the micro-devices includes velocity slip and temperature jump was investigated by Tunc and Bayazitoglu (2002). Adesanya (2015) studied free convection flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump and pointed out that an increase in slip parameter increased the flow velocity and decreased the shear stress at the suction wall.

In all the above aforementioned studies, the effects of radiative heat transfer on the temperature field were neglected. However, Viskana (2014) reported that the radiative heat transfer on the porous channel of small thickness can be treated as an effective diffusion process. Mahmoundi (2011) reported that ignoring the effects of thermal radiation on the porous wall channel lead to substantial error in the prediction of micro-flow devices. Therefore, it is necessary to consider the effects of thermal radiation in micro devices. The present study deals with the effects of thermal radiation on convection flow of heat generating fluid in a porous channel with velocity slip and temperature jump. The diffusion of radiation heat transfer was used and considered the Roseland approximation efficient of optically thickness.

Mathematical formulation

Unsteady laminar free convective flow of viscous incompressible heat generating fluid in a porous vertical channel with velocity slip and temperature in the presence of thermal radiation is considered. The micro- channel walls aretaken vertically and parallel to x-axis at $h = \pm h$. It is assumed that on wall $(y = h)$ the fluid is injected into the channel with constant velocity v_0 and sucked off at the wall $h = -h$ at the same constant velocity. It is also assumed that gas molecules are interacting with surface of the walls through force of attraction. Consequently, the molecules of gas are absorbed on the surface and reflected after some time lag. The time lag held to macroscopic velocity slip and temperature jump. The flow is induced by periodic heating at both walls. The equations governing the fluid flow and heat transfer are as follows;

$$\frac{\partial \bar{u}}{\partial t} - v_0 \frac{\partial \bar{u}}{\partial \bar{y}} = \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T - T_0) \quad \cdots\cdots\cdots\cdots (1)$$

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho c_P} \frac{\partial^2 T}{\partial \bar{y}^2} - \frac{\partial q}{\rho c_P \partial \bar{y}} + \frac{Q}{\rho c_P}(T_0 - T) \quad \cdots\cdots (2)$$

For rarefied flow with velocity slip and temperature jump, the relevant boundary conditions are;

$$\bar{u}(t,\bar{y}) = \frac{2-\xi}{\xi}\lambda\frac{\partial \bar{u}}{\partial \bar{y}},$$

$$T(t,\bar{y}) = T_1 - T_0 + T_2 e^{i\omega t}\left(\frac{2-\sigma_T}{\sigma_T}\frac{2\phi}{\phi+1}\frac{\lambda}{\Pr}\right)\frac{\partial T}{\partial \bar{y}} \quad \text{at} \quad \bar{y}=h,$$

$t>0$ .... (3)

$\bar{u}(t,\bar{y})=0$, $T(t,\bar{y}) = T_1 - T_0 + T_2 e^{i\omega t}$ at $\bar{y}=h$ $t>1$ .... (4)

Where: t is the time, $\bar{u}$ is the velocity, v_0 is constant horizontal velocity, υ is the kinematic viscosity, g is the acceleration due to gravity, β is the volumetric expansion, T is the fluid temperature, T_0, T_1 and T_2 are referenced fluid temperature respectively, k is the thermal conductivity, ρ is the fluid density, c_p is the specific heat at constant pressure, Q is the heat generation when $Q>0$ and the heat absorption when $Q<0$, q is the radiative heat flux, ξ is the tangent momentum accommodation, λ is the molecular mean free path, ϕ is the specific heat ratio and δ_T is thermal accommodation coefficient

Using Rosseland approximation, the radiative heat flux (Brewster, 1992) can be modeled as;

$$q = -\frac{4\sigma^* \partial T^4}{R\partial \bar{y}} \quad \text{........... (5)}$$

Where: σ^* is the Stefan-Boltzmann constant, R is the mean absorption coefficient. If the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature, the Taylor series T^4 for about T_0, after neglecting higher order terms, is given as;

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad \text{........... (6)}$$

Using equations (6) and (5) in equation (2), equation (2) becomes;

$$\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial \bar{y}} = \left(\frac{k}{\rho c_P} + \frac{16\sigma^* T_0^3}{3R\rho c_P}\right)\frac{\partial^2 T}{\partial \bar{y}^2} - \frac{\partial q}{\rho c_P \partial y} + \frac{Q}{\rho c_P}(T_0 - T) \quad \cdots (7)$$

Introducing the following perturbation techniques;

$$\left.\begin{aligned} \bar{u}(t,\bar{y}) &= (T_1 - T_0)u_0(y) + T_2 u_1(y)e^{i\omega t} \\ T(t,\bar{y}) &= T_0 + (T_1 - T_0)\theta_0(y) + T_2\theta_1(y)e^{i\omega t} \end{aligned}\right\} \quad \text{...... (8)}$$

With the following dimensionless parameters;

$$y = \frac{\bar{y}}{h},\ St = \frac{h^2\omega}{\upsilon},\ \Pr = \frac{\mu c_P}{k},\ s = \frac{hv_0}{\upsilon},\ \alpha = \frac{Qh^2}{k}$$

$$N = \frac{kR}{4\sigma^* T_0^3},\ Kn = \frac{\lambda}{k}\left(\frac{2-\sigma_T}{\sigma_T}\right)\frac{2\phi}{\phi+1},\ \gamma = \frac{(2-\xi)\lambda}{\xi h} \quad \text{......}$$

(9)

Using the perturbation techniques (8) and (9) in equations (1) and (7) with boundary conditions (3) and (4), the following steady and periodic equations are obtained for velocity and temperature;

$$u_0'' + su_0' + G\theta_0 = 0 \quad \text{........... (10)}$$

$$\left(1+\frac{4}{3N}\right)\theta_0'' + s\Pr\theta_0' - \alpha\theta_0 = 0 \quad \text{........... (11)}$$

$$\left.\begin{aligned} u_0(-1) &= \gamma u_0'(0), u_0(1) = 0 \\ \theta_0(-1) &= 1 + \frac{Kn}{\Pr}\theta_0'(-1), \theta_0(-1) = 1 \end{aligned}\right\} \quad \text{........... (12)}$$

$$u_1'' + su_1' - iStu_1 + G\theta_1 = 0 \quad \text{........... (13)}$$

$$\left(1+\frac{4}{3N}\right)\theta_1'' + s\Pr\theta_1' - i\Pr St - \alpha\theta_1 = 0 \quad \text{...... (14)}$$

$$\left.\begin{aligned} u_1(-1) &= \gamma u_1'(0), u_1(1) = 0 \\ \theta_1(-1) &= 1 + \frac{Kn}{\Pr}\theta_1'(-1), \theta_1(-1) = 1 \end{aligned}\right\} \quad \text{........... (15)}$$

Equations (13) and (14) are genersalised forms of equations (10) and (11) respectively. Therefore, whenever $St=0$, equations (10) and (11) are obtained from equation (13) and (14) respectively. Hence, the solution of the equations (13) and (14) when $St \neq 0$ are sufficient to described the periodic fluid flow behavior.

Method of solution is presented in this section. The exact solutions are obtained by using conditions (15) with equations (13) and (14) as follows;

$$\theta_1(y) = \frac{Be^{Ay}}{C} + \frac{Be^{-Ay}}{C} \quad \text{........... (16)}$$

$$u_1(y) = \frac{Fe^{Dy}}{HC} + \frac{Fe^{-Ey}}{HC} - \frac{GBe^{-Ay}}{C} \quad \text{........... (17)}$$

The rate of heat transfer (Nusselt number) is given by;

$$Nu = -\frac{\partial\theta}{\partial y} = -\frac{ABe^{Ay}}{C} + \frac{ABe^{-Ay}}{C} \quad \text{........... (18)}$$

$$Sf = \frac{\partial u}{\partial y} = \frac{DFe^{Dy}}{HC} + \frac{EFe^{-Ey}}{HC} - \frac{GBe^{-Ay}}{C} \quad \text{........... (19}$$

Results and Discussion

A series of computations were carried out for the effects of variation of the following parameters; Knudsen number (Kn), Navier slip parameter (γ), heat generation/absorption parameter (α), Grashof number (G) ,radiation parameter (N), suction/injection parameter (s), Strouhal number (St) and Prandtl number $\Pr$.

The following default values of the parameters; $Kn = 0.002$, $\gamma = 0.1$, $\alpha = 0.1$, $N = 0.1$, $s = 0.1$, $St = 0.1$, $G = 1$ and $\Pr = 0.1$ were used for the analysis of the fluid flow and heat transfer in the distribution profiles. All graphs and tables used the default values expect otherwise stated.

Fig. 1 presents the variation of Knudsen parameter with the temperature profiles. It can be seen that an increase in

FUW Trends in Science & Technology Journal ftstjournal@gmail.com
April, 2016 Vol. 1 No. 1 – e-ISSN: 24085162; p-ISSN: 20485170 pp 272-277

the Knudsen parameter decreases the temperature profiles. It is due to fact that the molecular distance of the fluid increases hence the heat flux decreases within the micro-channel. Fig. 2 depicts the different values of suction parameter $(s > 0)$ with temperature profiles. It is observed that the increasing in the suction parameter about enhancement in the temperature distribution profiles. Fig. 3 shows the influence of variation of injection parameter $(s < 0)$ on the temperature profiles. It can be seen that the temperature profile decreases with increasing in the value of injection parameter. Fig. 4 represents the variation of strouhal number with temperature profile. It is observed that the fluid temperature is decreasing with increasing (St). It is because the frequency of heating is increasing; thereby reduce the intensity of heat on the channel walls. Fig. 5 shows the variation of radiation parameter on the temperature profiles. It is observed that increasing radiation parameter thereby decreases the fluid temperature. Fig. 6 and 7 depict the effects of heat generation and absorption parameters on the temperature profiles. It can be seen that fluid temperature decreasing with heat generation/absorption parameter. This is due fact that temperature gradient are higher near the walls of the channel.

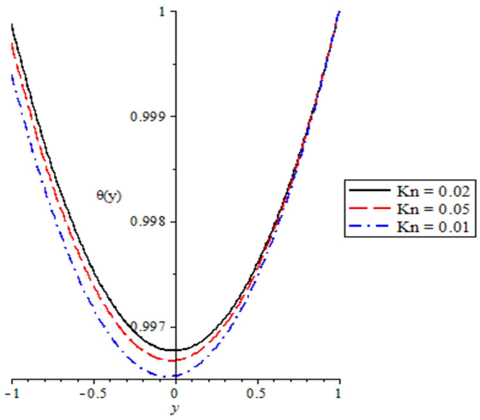

Fig. 1: Temperature profiles for diferent values of $s > 0$

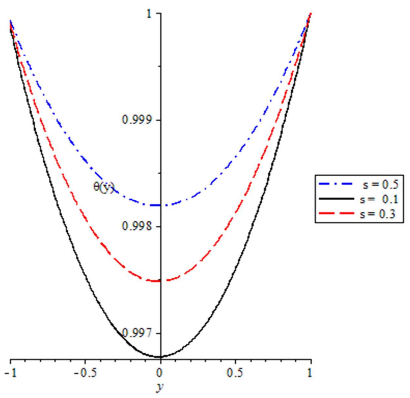

Fig. 2: Temperature profiles for diferent values of Kn

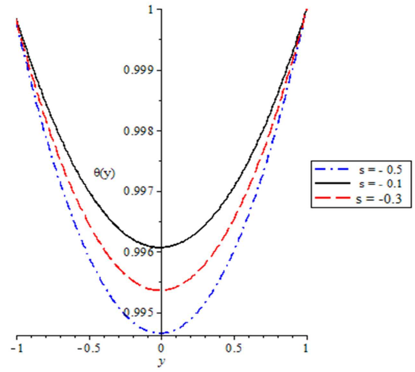

Fig. 3: Temperature profiles for diferent values of St

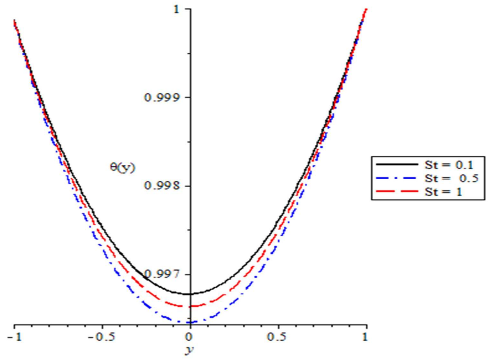

Fig .4: Temperature profiles for diferent values of $s < 0$

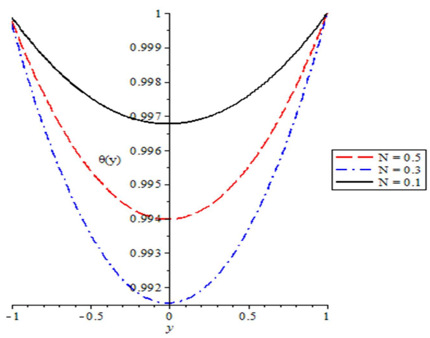

Fig. 5: Temperature profiles for diferent values of $\alpha > 0$

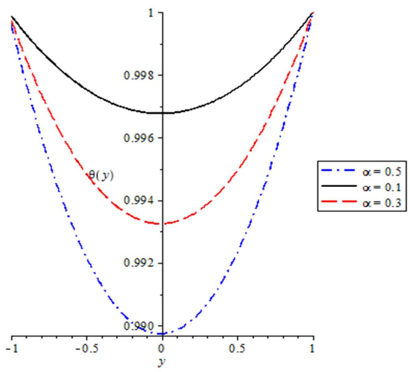

Fig. 6: Temperature profiles for diferent values of N

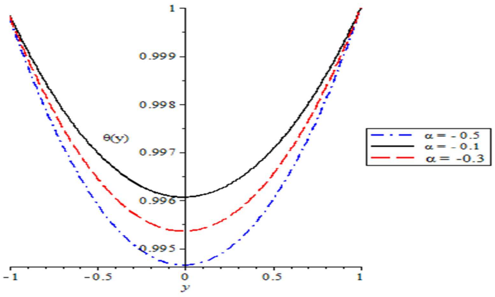

Fig. 7: Temperature profiles for diferent values of γ

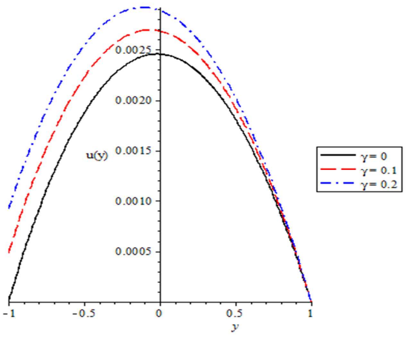

Fig . 8: Temperature profiles for diferent values of $\alpha > 0$

Fig. 8 illustrates the variation of Navier slip on the velocity profiles. It can be seen that increases in the Navier slip enhance the flow at the wall of the channel. As a result, the molecules of gas interact with the suction wall. Fig. 9 and 10 present the influence of suction/injection parameter on the velocity profiles. It is observed that an increase in suction/injection enhances the velocity of the fluid motion throughout the channel. Fig. 11 shows the different values of Strouhal number with velocity profiles. It is observed that an in increase Strouhal number cause a reduction in the velocity profiles. Fig. 12 represents the variation of radiation on the velocity profiles. It can be seen that the velocity profiles increases as the radiation parameter increases. This is because the intensity of heat produced through thermal radiation increases, by brake the bond holding the component of the fluid particles.

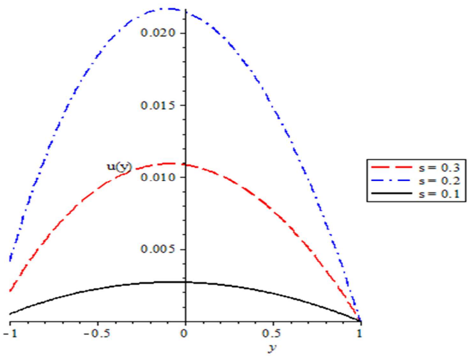

Fig. 9: Velocity profiles for diferent values of $s < 0$

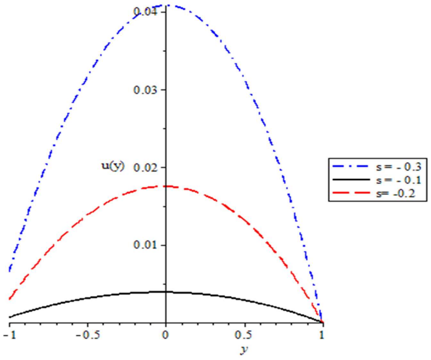

Fig. 10: Velocity profiles for diferent values of $s > 0$

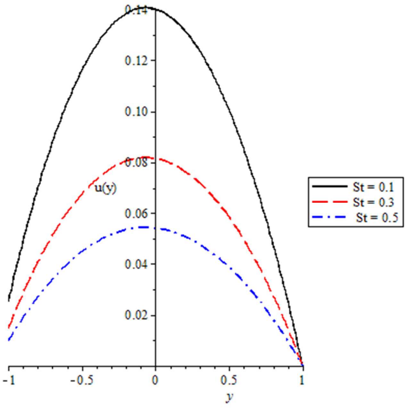

Fig. 11: Velocity profiles for diferent values of N

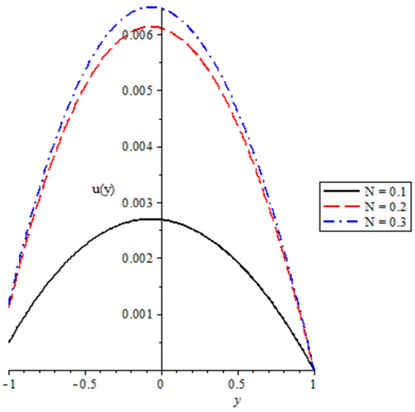

Fig. 12: Velocity profiles for diferent values of St

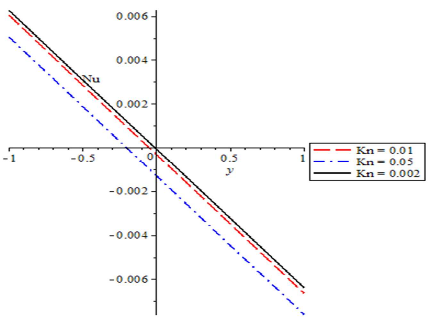

Fig. 13: Nusselt profiles for diferent values of N

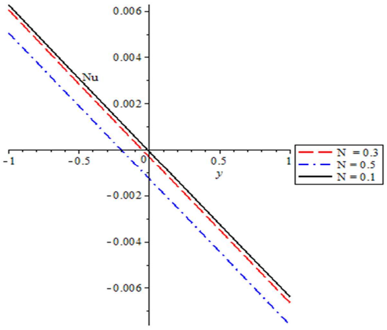

Fig. 14: Nusselt profiles for diferent values of Kn

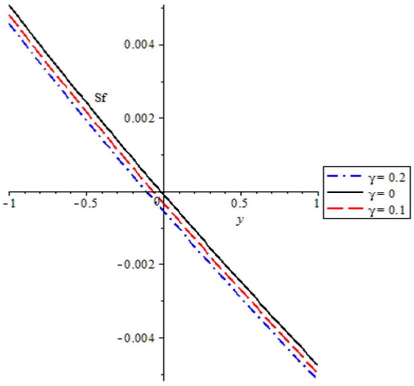

Fig. 15: Skin friction profiles for different values of γ

Fig. 13 depicts the effects of Knudsen number on the rate of heat transfer at the walls with suction/injection. It is observed that an in increase Knudsen decreases the rate of heat transfer at the walls. Fig. 14 shows increasing radiation parameter causes a reduction on the Nusselt number profiles. Fig. 15 depicts the influence of Navier slip on the skin friction profiles. It can be seen that increasing Navier slip parameter weakens the skin friction profiles.

Conclusion

The effects of Knudsen number, radiation, Navier slip, suction/injection and heat generation/absorption parameters were investigated. The exact solutions for temperature, velocity, Nusselt number and skin friction were obtained. It was observed that Knudsen number, Strouhal number, radiation , injection and heat generation/absorption parameters decreased the fluid temperature while the temperature of the fluid increased with increases in suction parameter. The velocity profiles increased with an increases in any of radiation, Navier slip and suction/injection parameter wheareas Strouhal number decreased the velocity profiles. Nusselt number decreased with increasing in the value of radiation parameter or Knudsen number and skin friction decreased with an increase in Navier slip parameter.

References

Adesanya SO 2015. Free convective flow of heat generating fluid through a porous vertical channel with velocity slip and temperature jump. *Ain Shams Eng. J.*, 10: 10 – 16.

Brewster MQ 1992. *Thermal Radiative Transfer and Properties*. New York. John Wiley & Sons.

Dehghan M & Tabrizi HB 2014. Turbulence effects on the grannular model of particule motion in a boundary layer flow. *Can. J. Chem. Eng*., 92: 189-195.

Dharaiya VV & Kandlikar SG 2012. Numerical investigation of heat transfer in rectangular micro-channel under H_2 bound any condition during developing and fully development laminar flow. *J. Heat Transfer* 134: 1-10.

Hung WC & Ru Y 2006. A numerical study for slip flow heat transfer. *Appl. Math. Comp*. 173: 1246-1264.

Jha BY & Ajibade AO 2009. Free convective flow of heat generation/ absorption fluid between vertical porous plates with periodic heat input. *Int. Commun. Heat Mass Transfer*, 36: 624-631.

Jha BY & Ajibade AO 2012. Effects of viscous dissipation on natural convective flow between vertical parallel plates with time-periodic boundary conditions. *Commun Nonlinear Sci. Numer. Simulat*, 17: 1576-1587.

Mahmoudi Y & Maerefat M 2011. *Analytical investigation of heat transfer ehancement in a channel partially filled with a porous material under local non-equlibrium condition*. *Int. J. Therm. Sci.*, 50(12): 2386-2401.

Tunc G & Bayaziglu Y 2002. Heat transfer in rectangular micro-channel. *Int. J. Heat Mass Transfer*, 78: 765-773.

Vafai K & Zhu L 1999. Analysis of two layered micro-channel heat sink concept in electronic cooling. *Int. J. Heat Mass Transfer*, 42: 2287-2287.

Vafai K & Khaled HRA 2005. Analysis of flexible micro-channel heat sink systems. *Int. J. Heat Mass Transfer*, 48: 1739-1746.

Viskana R 2014. Overview of radiative transfer models. *Int. J. Mass Transfer*, 78: 7-16.

Wang CY 1988. Free convective between vertical plates with periodic heat input. *ASME J. Heat Transfer*, 110: 508-511.

Appendix l

$$A=\frac{\sqrt{3N}\sqrt{-s\Pr+\alpha+iSt\Pr}}{\sqrt{3N+4}},\ m_1=\sqrt{N}\sqrt{-s\Pr+\alpha+iSt\Pr}+\Pr\sqrt{3N+4}$$

$$m_2=\Pr\sqrt{3N+4},\ B=K_n\sqrt{3}e^{A}m_1e^{A}-m_2e^{A}$$

$$m_3=\sqrt{N}\sqrt{-s\Pr+\alpha+iSt\Pr},\quad m_4=K_n\sqrt{3N}\sqrt{-s\Pr+\alpha+iSt\Pr}\ C=K_n\sqrt{3}e^{A^2}m_3+m_4e^{A^2},$$

$$D=-\frac{1}{2}s+\frac{1}{2}\sqrt{s^2+4ist},\quad D=-\frac{1}{2}s-\frac{1}{2}\sqrt{s^2+4ist}$$

$$m_5=\sqrt{s^2+4iSt},\ m_6=e^{\frac{1}{2}s+\sqrt{s^2+4iSt}},\ m_7=e^{2A}e^{-A}$$

$$\begin{aligned}F=\Big(&-A\,m_5\,m_6m_7\gamma S-2\,\mathrm{I}\,A\,m_7\,m_6\gamma St-\mathrm{I}\,m_5m_6m_7\gamma St-\mathrm{I}\,m_5m_6\,\gamma S\,St-2\,m_8\,m_9\gamma+2\,\mathrm{I}\,m_9\,e^{A}\,St\\&-2\,\mathrm{I}\,e^{-A}\,m_9\,St+2\,m_8\,e^{A}\,m_9-2\,m_8\,\gamma\,m_9+m_5m_6m_7+m_7\,m_9\,\gamma S+A\,m_6\,\gamma S^2+2\,\mathrm{I}\,m_9m_6St\\&-2\,\mathrm{I}\,m_6m_7\,St+2m_6\,m_9\,S-2\,A\,m_6m_7\,S-2\,m_6m_7+m_5m_6\,m_7\,\gamma+m_6m_7\,\gamma S-Am_6m_7\,\gamma S^2\\&+A\,m_5m_6\,\gamma S+2\,\mathrm{I}\,A\,m_6m_9e^{A}\,\gamma St-\mathrm{I}\,e^{-A}\,m_5\,m_6\gamma St-\mathrm{I}\,e^{-A}\,m_6\,\gamma S\,St-2\,A^2\,m_6m_7+2\,A^2\,m_{(}\\&-2\,A^2\,m_9\,e^{-2A}\,e^{A}+2\,m_6m_7-2\,A\,m_6\,e^{A}\,S+2\,A\,e^{-A}\,m_9\,S\Big)\,G\end{aligned}$$

$$m_{10}=e^{-\frac{1}{2}S+\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}\,e^{\frac{1}{2}S+\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}\sqrt{S^2+4\,\mathrm{I}\,St},\quad m_{12}=e^{-\frac{1}{2}S+\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}$$

$$m_{11}=e^{\frac{1}{2}S-\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}\sqrt{S^2+4\,\mathrm{I}\,St}$$

$$m_{12}=e^{-\frac{1}{2}S+\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}$$

$$m_{13}:=e^{-\frac{1}{2}S-\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}\,e^{\frac{1}{2}S-\frac{1}{2}\sqrt{S^2+4\,\mathrm{I}\,St}}$$

$$\begin{aligned}H=\Big(&\gamma A^4\,m_{10}+\gamma A^4\,m_{11}\,S+\gamma A^4\,m_{10}-\gamma A^4\,m_{11}\,S-4\,\mathrm{I}\,A^2\,m_{11}\,St-\gamma A^2\,m_{10}\,S^2\\&-2\,\mathrm{I}\,\gamma\,m_{11}\,S\,St-\gamma A^2 m_{11}\,S^3-2\,\mathrm{I}\,\gamma A^2\,m_{10}\,St-\gamma A^2\,m_{10}\,S^2+2\,\mathrm{I}\,\gamma A^2\,m_{11}\,S\,St\\&+\gamma A^2\,m_{11}\,S^3-\gamma\,m_{10}\,St^2-\gamma\,m_{11}\,S\,St^2-\gamma\,m_{10}\,St^2+\gamma\,m_{11}\,S\,St^2+2\,A^4\,m_{11}-2\,A^4 m_{11}\\&+4\,\mathrm{I}\,A^2\,m_{13}\,St-2\,A^2\,m_{13}\,S^2-2\,\mathrm{I}\,\gamma A^2\,m_{10}\,St+2\,A^2\,m_{10}-2\,m_{11}\,St^2+2\,m_{13}St\Big)\end{aligned}$$